Worksheet A: Introduction to Sequences

For each sequence, find the next 4 terms.

1. 1, 2, 4, 7, 11, ...

2. 3, 9, 27, 81, ...

3. 1, 3, 7, 15, 31, ...

4. 192, -96, 48, -24, ...

5. 2, 6, 12, 20, 30, ...

6. \( \frac{1}{2}, \frac{1}{4}, \frac{1}{8}, \frac{1}{16}, \ldots \)

7. \( 1, \frac{1}{4}, \frac{1}{9}, \frac{1}{16}, \frac{1}{25}, \ldots \)

8. S, M, T, W, T, ...

9. J, F, M, A, M, ...

10. \[ \bigcirc \quad \bigcirc \quad \bigcirc \quad \bigcirc \\]

11. \[ \square \quad \bigcirc \quad \bigcirc \\]
Worksheet B: nth term of a sequence

Find the nth term in each sequence

1. 

<table>
<thead>
<tr>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
<th>...</th>
<th>n</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>1</td>
<td>2</td>
<td>3</td>
<td>4</td>
<td>5</td>
<td></td>
<td>?</td>
</tr>
</tbody>
</table>

2. 

<table>
<thead>
<tr>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
<th>...</th>
<th>n</th>
</tr>
</thead>
<tbody>
<tr>
<td>4</td>
<td>7</td>
<td>10</td>
<td>13</td>
<td>16</td>
<td>19</td>
<td></td>
<td>?</td>
</tr>
</tbody>
</table>

3. Study the pattern and then complete the exercise based on your observations

<table>
<thead>
<tr>
<th>2</th>
<th>= 2</th>
</tr>
</thead>
<tbody>
<tr>
<td>2 + 4</td>
<td>= 6</td>
</tr>
<tr>
<td>2 + 4 + 6</td>
<td>= 12</td>
</tr>
<tr>
<td>2 + 4 + 6 + 8</td>
<td>= 20</td>
</tr>
<tr>
<td>2 + 4 + 6 + 8 + 10</td>
<td>= 30</td>
</tr>
<tr>
<td>2 + 4 + 6 + 8 + 10 + 12</td>
<td>= 42</td>
</tr>
<tr>
<td>2 + 4 + ... + 12 + 14</td>
<td>= ____</td>
</tr>
<tr>
<td>2 + 4 + ... + 14 + 16</td>
<td>= ____</td>
</tr>
<tr>
<td>2 + 4 + ... + 16 + 18</td>
<td>= ____</td>
</tr>
<tr>
<td>2 + 4 + ... + 18 + 20</td>
<td>= ____</td>
</tr>
</tbody>
</table>

Conjecture: the sum of the first 30 even numbers is _____________________

Use inductive reasoning to find the NEXT TERM of each sequence

4. 4, 5, 7, 11, 19, 35, 67, ____

5. 16, 2, 8, 18, 2, 9, 20, 2, ____
Worksheet C: Explicit Formulas - Factoring

1. \[ \begin{array}{ccccccccc} & 1 & 2 & 3 & 4 & 5 & 6 & \ldots & n \\ 0 & 3 & 8 & 15 & 24 & 35 & \ldots & \\ \end{array} \]

2. \[ \begin{array}{ccccccccc} & 1 & 2 & 3 & 4 & 5 & 6 & \ldots & n \\ 12 & 25 & 42 & 63 & 88 & 117 & \ldots & \\ \end{array} \]

3. \[ \begin{array}{ccccccccc} & 1 & 2 & 3 & 4 & 5 & 6 & \ldots & n \\ 12 & 20 & 30 & 42 & 56 & 72 & \ldots & \\ \end{array} \]

4. \[ \begin{array}{ccccccccc} & 1 & 2 & 3 & 4 & 5 & 6 & \ldots & n \\ 0 & 0 & 2 & 6 & 12 & 20 & \ldots & \\ \end{array} \]
1. Fill in the blanks to Pascal’s Triangle (above)
2. In the sequence 1, 1, 1, 1, … what is the nth term?
3. The sequence in the second diagonal is 1, 2, 3, 4, ... what is the nth term?
4. Fill in the chart:

<table>
<thead>
<tr>
<th>Number of the row</th>
<th>0</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
<th>...</th>
<th>n</th>
</tr>
</thead>
<tbody>
<tr>
<td>Sum of the numbers in the row</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
There are many other triangular arrays of numbers similar to Pascal’s Triangle that have interesting patterns in them.

7. Find the terms in the next row of the triangular array.

```
   1
  1  1
 1  3  1
 1  5  5  1
 1  7 13  7  1
 1  9 25 25  9  1
    ___  ___  ___  ___  ___  ___  ___
```

8. What is the sum of the numbers in the 500th row of the triangular array below?

```
   1
  1  2  1
  1  2  3  2  1
  1  2  3  4  3  2  1
  1  2  3  4  5  4  3  2  1
  1  2  3  4  5  6  5  4  3  2  1
```
Worksheet E: Recursively Defined Sequences

Find the first five terms of these recursively defined sequences

1. \( a_1 = 2, a_{n+1} = 4a_n - 3 \)

2. \( a_1 = -3, a_{n+1} = 2a_n - 5 \)

3. \( a_1 = 8, a_{n+1} = \frac{1}{2}a_n + 2 \)

4. \( a_1 = 3, a_2 = 3, a_{n+1} = a_n - a_{n-1} \)
Name ____________________________ Date ________________

Worksheet F

Unit 1 - Quiz A Practice ___/50

1-5: Find the next term in a numeric sequence (1 pts each blank)

1. 0, 10, 21, 33, 46, 60, ______
2. 1/2, 1/4, 1/8, 1/16, ______
3. 20, 18, 16, 14, ______
4. 1, 1, 2, 3, 5, 8, 13, ______
5. 1, 8, 27, 64, ______

6. Fill in the blanks.
   12345679 * 9 = 11111111
   12345679 * 18 = 22222222
   12345679 * 27 = 33333333
   12345679 * 36 = __________
   12345679 * ___ = 55555555

7-9 Find an expression for the nth term (3 pts each)

7. 

    | 1 | 2 | 3 | 4 | 5 | ... | n |
    |---|---|---|---|---|-----|---|
    | -2| -1|  0|  1|  2| ... | ? |

  nth term = __________

8. 

    | 1 | 2 | 3 | 4 | 5 | ... | n |
    |---|---|---|---|---|-----|---|
    |  4|  8| 12| 16| 20| ... | ? |

  nth term = __________

9. 

    | 1 | 2 | 3 | 4 | 5 | ... | n |
    |---|---|---|---|---|-----|---|
    |  2|  5|  8| 11| 14| ... | ? |

  nth term = __________

10-11 Factor to find an expression for the nth term (3 pts each)

10. 

    | 1 | 2 | 3 | 4 | 5 | ... | n |
    |---|---|---|---|---|-----|---|
    |  3|  8| 15| 24| 35| ... | ? |

  nth term = __________

11. 

    | 1 | 2 | 3 | 4 | 5 | ... | n |
    |---|---|---|---|---|-----|---|
    |  0|  5| 12| 21| 32| ... | ? |

  nth term = __________
12. Fill in the blanks using patterns in Pascal’s Triangle. (1 pt each = 12 pts total)

\[
\begin{array}{cccccccccc}
1 \\
1 & 1 \\
1 & 2 & 1 \\
1 & 3 & 3 & 1 \\
1 & 4 & 6 & 4 & 1 \\
1 & 5 & 10 & 10 & 5 & 1 \\
1 & 6 & 15 & 20 & 15 & 6 & 1 \\
1 / 21 & 35 & 35 & 21 & 7 & 1 \\
\end{array}
\]

13–15 Generate terms of the recursively-defined sequences (5 pts each)

13. Find the first 5 terms of the recursively-defined sequence
   \( a_1 = 4; \quad a_{n+1} = 8a_n \)
   
   \( a_2 = \)
   
   \( a_3 = \)
   
   \( a_4 = \)
   
   \( a_5 = \)

14. Find the first 5 terms of the recursively-defined sequence
   \( a_1 = 0; \quad a_{n+1} = a_n + 13 \)
   
   \( a_2 = \)
   
   \( a_3 = \)
   
   \( a_4 = \)
   
   \( a_5 = \)

15. Find the first 5 terms of the recursively-defined sequence
   \( a_1 = 6; \quad a_{n+1} = 5a_n - 3 \)
   
   \( a_2 = \)
   
   \( a_3 = \)
   
   \( a_4 = \)
   
   \( a_5 = \)

16. What is your favorite cereal? ___________ (1 pt)
Worksheet G: Introduction to Series

The general term of a sequence is given. In each case find the first four terms, the $10^{th}$ term and the $15^{th}$ term.

1. $a_n = 3n + 1$
2. $a_n = n^2 + 1$

3. $a_n = n^2 - 2n$
4. $a_n = \left(\frac{-1}{2}\right)^{n-1}$

For each sequence, find the “rule”

5. 3, 9, 27, 81, 243,....
6. $\sqrt{2}, \sqrt{4}, \sqrt{6}, \sqrt{8}, \sqrt{10}$

7. 1•2, 2•3, 3•4, 4•5,....

Find $S_1, S_2, S_3$ and $S_4$ for each sequence

8. $\frac{1}{3}, \frac{1}{6}, \frac{1}{12}, \frac{1}{24}, \frac{1}{48}$,....

9. -1, 3, -5, 7, -9,....

10. 4, 7, 10, 13, 16, ....

11. -3, 9, -27, 81, -243, ...
Worksheet H: Sums and Sigma Notation

Rename and evaluate each sum

1. \( \sum_{n=1}^{5} \frac{1}{2n} \)

2. \( \sum_{n=1}^{6} \frac{1}{2n+1} \)

3. \( \sum_{n=1}^{5} 2^n \)

4. \( \sum_{n=4}^{7} \sqrt{2n-1} \)

5. \( \sum_{n=0}^{4} \pi n \)

Write the following in sigma notation

6. \( \frac{1}{2} + \frac{2}{3} + \frac{3}{4} + \frac{4}{5} + \frac{5}{6} + \frac{6}{7} \)

7. 3 + 6 + 9 + 12 + 15

8. -2 + 4 - 8 + 16 - 32 + 64

9. \( \frac{1}{1^2} + \frac{1}{2^2} + \frac{1}{3^2} + \frac{1}{4^2} + \frac{1}{5^2} \)

10. 4 - 9 + 16 - 25 + …

11. 9 - 16 + 25 - 36 + 49 - 64 + …
Worksheet I: More Sums and Sigma Notation

1. Find an expression for the value of the $n$th term:

   $3, 5, 7, 9, 11, 13, ...$

2. Find an expression for the value of the $n$th term:

   $0, 8, 18, 30, 44, ...$

3. Explain the significance of the $(-1)^n$ in the expression $\sum_{n=1}^{4} (-1)^n (4n + 3)$.

   ____________________________________________________
   ____________________________________________________
   ____________________________________________________

4. Explain the significance of the $(-1)^{n+1}$ in the expression $\sum_{n=1}^{4} (-1)^{n+1} (4n + 3)$.

   ____________________________________________________
   ____________________________________________________
   ____________________________________________________

OVER
5. Express the series $1 + 7 + 13 + 19 + 25 + 31 + 37$ using sigma notation.

6. Express the series $3 - 5 + 7 - 9$ using sigma notation.

7. Express the series $15 + 28 + 45 + 66 + 91$ using sigma notation.
Worksheet J: Arithmetic Sequences

Based on the terms given, state whether or not each sequence is arithmetic. If it is identify the common difference, d.

1. 6, 10, 14, 18, 22, ...
2. 8, 5, 2, -1, -4, ...
3. 5, -5, 5, -5, 5, -5, ...
4. 9, 7, 5, 3, 1, ...
5. 3, 6, 12, 24, ...
6. -1, 1, -1, 1, ...
7. 0, $\frac{1}{2}$, 1, $\frac{3}{2}$, 2, ...
8. $\frac{2}{7}$, $\frac{4}{7}$, 1, $\frac{11}{7}$, $\frac{16}{7}$, ...
9. -2.8, 3.9, 5.0, 6.1, 12.2
10. $\pi$, 2$\pi$, 3$\pi$, 4$\pi$, 5$\pi$, ...
Worksheet K: Geometric Sequences

Determine whether each sequence is a geometric sequence. If so, identify the common ratio, r, and give the next 3 terms.

1. 3, 6, 12, 24, ...
2. 2, 6, 18, 54, ...

3. 9, 3, 1, \(\frac{1}{3}\), ...
4. 2, 4, 6, 8, ...

5. 1, -4, 16, -64, ...
6. 2, 3, 4.5, 6.75, ...

7. 10, 2, \(\frac{2}{5}\), \(\frac{2}{25}\), ...
8. 6, 42, 294, ...

9. 16, 20, 25, 31.25, ...
Worksheet L

Decide whether each sequence is arithmetic, geometric, or neither. State the common difference or the common ratio if it exists.

1. 4, 10, 16, 22, 28, ...

2. 5, 8, 11, 14, ...

3. 4, 12, 36, 108, ...

4. 2, -4, 8, -16, ...

Expand the following and evaluate

5. \( \sum_{n=1}^{4} 4n \)

6. \( \sum_{n=2}^{6} 2n + 1 \)

Express using sigma notation

7. a. -1 + 4 - 9 + 16 - 25
   b. -1 + 4 - 16 + 64

8. \( \frac{1}{9} + \frac{1}{16} + \frac{1}{25} + \frac{1}{36} \)
9. Fill in the blanks

\[
\begin{array}{cccc}
1 & 1 & 2 & 1 \\
1 & 2 & 2 & 1 \\
1 & 3 & 3 & 1 \\
1 & 4 & 6 & 4 & 1 \\
\end{array}
\]

Find \(a_2, a_3, a_4, a_5\) and \(a_6\) for each of the following recursive rules

10. \(a_1 = 2\)
\(a_n = a_{n-1} + 3\)

\(a_2 = \) ________
\(a_3 = \) ________
\(a_4 = \) ________
\(a_5 = \) ________

11. \(a_1 = 5\)
\(a_{n+1} = 2a_n - 1\)

\(a_2 = \) ________
\(a_3 = \) ________
\(a_4 = \) ________
\(a_5 = \) ________

Find an explicit rule for each of the following

12.

<table>
<thead>
<tr>
<th>(n)</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>...</th>
<th>(n)</th>
</tr>
</thead>
<tbody>
<tr>
<td>(a_n)</td>
<td>4</td>
<td>5</td>
<td>6</td>
<td>7</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

13.

<table>
<thead>
<tr>
<th>(n)</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>...</th>
<th>(n)</th>
</tr>
</thead>
<tbody>
<tr>
<td>(a_n)</td>
<td>-6</td>
<td>-4</td>
<td>0</td>
<td>6</td>
<td>14</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>